Three flavor neutrino oscillations, LSND, SNP and ANP

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Abstract

The recently reanalysed LSND data is investigated in the context of three flavor oscillations, with an emphasis on mass hierarchies and s_{13}. The resultant mass hierarchies and oscillation angles are tested with regard to "key" features of solar neutrinos and atmospheric neutrinos, e.g., "average" survival probability in the case of solar neutrinos, and zenith angle dependence and up-down asymmetry in the case of high energy atmospheric neutrinos. We find there are three distinct mass hierarchies, e.g., $\Delta m << \Delta M, \Delta m < \Delta M$ and $\Delta m \simeq \Delta M$. In the first and second case, the calculated range of s_{13} is in agreement with the "LMA" solution of Akhmedov *et al.*, the lower limit on s_{13} in these cases is also in agreement with the recent analysis of Garcia *et al.* based on the constraints of SNP, ANP and CHOOZ, therefore, strongly supporting the neutrino oscillations observed at LSND. Further, the solutions of s_{13} found in the third case correspond to the value of s_{13} found by Akhmedov *et al.* in the case of "SMA" and "LOW" solutions. A rough estimate of the possibility of the existence of CP violation in the leptonic sector is also carried out for different possible ranges of s_{13}, indicating that the CP asymmetries may be measureable even in the case of LSND.

Neutrino oscillations provide the preferred solution for the anomalous behaviour observed in the case of solar neutrinos [1]-[7], atmospheric neutrinos [8]-[14], as well as at LSND [15]. This essentially implies that the neutrinos are massive particles and the observed flavor eigenstates are linear combinations of mass eigenstates, in parallel to the quark mixing phenomenon. By far, the strongest evidence for neutrino oscillations is provided by the SuperKamiokande(SK) atmospheric neutrino data [9]. In view of the fact that

the signal for neutrino oscillations is also the signal for physics beyond the Standard Model (SM) [16, 17, 18], an intense amount of activity [19]-[22], both at the experimental as well as at the phenomenological level, is going on to fix the mass hierarchies as well as the parameters of the neutrino oscillations.

Several detailed and exhaustive analyses [23] have been carried out in the case of the solar neutrino data as well as in the case of the atmospheric neutrino data. These analyses along with the results from several other experiments have provided valuable information about the masses as well as about the mixing parameters. The constraints on masses and mixings are presented in terms of the mixing angles $(\theta_{12}, \theta_{13}, \theta_{23})$, defined analogous to the quark mixing angles [24], and the mass square differences. The most important constraint is provided by the SK analysis of their atmospheric neutrino data, [9, 25, 26], for example,

$$\Delta m_{32}^2 \simeq (2-6)\times 10^{-3} eV^2, \sin\theta_{23} \simeq 0.545 - 0.839, \tag{1}$$

where subscripts '2' and '3' correspond to ν_μ and ν_τ. Although the constraint is for two flavor oscillations, however, in view of the CHOOZ constraint [27]

$$\sin^2\theta_{13} \equiv |U_{e3}|^2 \leq (0.06 - 0.018) \text{ for } \Delta m_{31}^2 = (1.5-5)\times 10^{-3} eV^2, \tag{2}$$

it remains valid in the case of three flavor oscillations also. It is generally believed that the bimaximal mixing matrix [28] provides a good approximation to the neutrino mixing matrix and is also consistent with the recent most exhaustive three flavor analysis of Solar Neutrino Problem (SNP) and Atmospheric Neutrino Problem (ANP) by Garcia *et al.* [26].

It is widely believed that the oscillation solutions to SNP, ANP and LSND data requires three different mass scales [29], therefore a simultaneous solution seems to be possible only within four flavor(3 active + 1 sterile) scenario [29]. However, the latest analysis of the SK data [5] strongly indicates the absence of the role of sterile neutrinos in understanding SNP and ANP. This has complicated the issues related to neutrino mass scales. Besides this, there are several issues where more information is needed to deepen the understanding of the phenomenology of neutrino oscillations. One of the important issue is to find the lower limit of s_{13}, the upper limit, of course, is provided by the CHOOZ constraint. Some attempts have been made in this direction [26, 30], however a clear picture is yet to emerge. Similarly, despite considerable progress in understanding the neutrino mass differences, more information is very much desirable in knowing the various mass hierarchies which can fit the solar neutrino data, the atmospheric neutrino data and the LSND data simultaneously. The other burning issue, in the context of neutrino oscillations, is the existence or non existence of CP violation in the leptonic sector, for which serious effort is going on to decipher it in the Long BaseLine(LBL) experiments. Any

clue about CP violation, from phenomenological considerations, would have far reaching consequences for the neutrino oscillation phenomena as well as for the planned LBL experiments and Neutrino Factories [20, 21, 22, 31].

The reaffirmation of the LSND results presented at Neutrino 2000 as well as the lowering of their probability, would have important consequences as this is an appearance experiment, unlike solar neutrino experiments and atmospheric neutrino experiments. Another extremely important observation which has been emphasized at Neutrino 2000 is the absence of energy dependence of solar neutrino recoil energy spectrum by SK [5] as well as by Sudbery Neutrino Observatory (SNO) [7]. Similarly, the preliminary observations of K2K experiment [32] as well as of SNO endorse the conclusions of SK regarding the hypothesis of neutrino oscillations. In view of the recent observations of SK about sterile neutrinos as well as the issue of three different mass scales being required to explain SNP, ANP and LSND results, the question regarding the number of neutrino species has become extremely important. In this context, Minakata [31] has strongly advocated that one should try to re-explore, within three active flavors, the crucial features of the solar neutrino data, the atmospheric neutrino data and the LSND data which can be reconciled simultaneously. Some preliminary attempts [33, 34, 35] have been made in this connection which need to be updated in the context of the observations at Neutrino 2000. Further, these attempts have not gone into the details of the mixing angles, in particular the lower limit of s_{13} and its implications on CP violation. Furthermore, these analyses do not adequately emphasize the issues related to flux uncertainties as well as up-down asymmetry in the case of atmospheric neutrinos.

The purpose of the present communication, on the one hand, is to explore, within three active flavors, the possibility of reconciling the latest LSND data with some of the "key" features of solar and atmospheric neutrino data. On the other hand, it is to explore the constraints on the element s_{13}, the resultant mass hierarchies as well as the possible clues about the order of CP violation in the leptonic sector.

Before we proceed further, we underline the "key" features of the solar neutrinos and the atmospheric neutrinos which are relevant to the present investigation. In the case of solar neutrinos, the most important feature of the data is the ratio of flux of solar neutrinos observed by various ongoing experiments to the flux predicted by Standard Solar Model(SSM) [36]. In view of the observation at SK and SNO about the energy independence of the solar neutrino flux, it is expected that the above mentioned ratio should be same in all the solar neutrino experiments. In this situation, as advocated by Bahcall *et al.* [36], and Conforto *et al.* [34], one can consider the "average"

solar survival probability as,

$$P_{sol} = 0.50 \pm 0.06. \tag{3}$$

Similarly, in the case of atmospheric neutrinos the "key" aspects of the data which indicate the possibility of neutrino oscillations are zenith angle dependence of the ratio (observed versus Monte Carlo (MC)) of the neutrino fluxes and the up-down asymmetry. In view of the fact that the ratios are very much dependent on the MC simulations of neutrino fluxes, it is perhaps desirable to concentrate on those features of the data wherein the flux uncertainties are minimum. Details in this regard will be discussed later in the text.

To begin with, we consider the neutrino mixing matrix,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{4}$$

where ν_e, ν_μ, ν_τ are the flavor eigenstates and ν_1, ν_2, ν_3 are the mass eigenstates. In the PDG representation [24], the mixing matrix can be expressed as,

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}. \tag{5}$$

In the sequel, we detail the relevant appearance and disappearance probabilities in terms of masses and mixing angles pertaining to the LSND experiment, the SNP and the ANP. In the case of LSND experiment, we express the probability in terms of the mixing matrix. The expression for P_{LSND}, ignoring CP violation effects, can be expressed as [37]

$$\begin{aligned} P_{LSND} \;\; = \;\; & -4U_{21}U_{11}U_{22}U_{12}\sin^2\left(\frac{\Delta m^2 L}{4E}\right) \\ & -4U_{21}U_{11}U_{23}U_{13}\sin^2\left(\frac{(\Delta m^2 + \Delta M^2)L}{4E}\right) \\ & -4U_{22}U_{12}U_{23}U_{13}\sin^2\left(\frac{\Delta M^2 L}{4E}\right), \end{aligned} \tag{6}$$

where $\Delta m^2 \equiv \Delta m^2_{21} = m_2^2 - m_1^2$, $\Delta M^2 \equiv \Delta m^2_{32} = m_3^2 - m_2^2$, L denotes the neutrino flight path *i.e.* the distance between the neutrino source and the dectector and E is the average energy of the neutrinos. If $\Delta m^2, \Delta M^2$ are expressed in eV^2 and L is expressed in metres then E is in MeV. The LSND set up is characterised by L = 30m and $36MeV < E < 60MeV$. For the

purpose of calculations, we take the average energy $E = 42MeV$ and from their latest analysis [15], we have

$$P_{LSND} = (2.5 \pm 0.6 \pm 0.4) \times 10^{-3} \text{ with } \Delta m^2 > 0.2eV^2. \tag{7}$$

It is to be noted that when the above constraint is combined with the limit provided by the CDHSW data [38], the range of Δm^2_{LSND} becomes

$$0.2eV^2 < \Delta m^2_{LSND} < 0.4eV^2. \tag{8}$$

In view of the fact that s_{12} and s_{23} are fairly well known, one can make use of the equation (6), with the identification of ΔM^2 with Δm^2_{LSND}, to find the element s_{13} for different mass hierarchies, however these values have to satisfy the "key" features of the solar neutrinos as well as the atmospheric neutrinos.

To this end, we first consider the case of solar neutrinos. The survival probability of solar electron neutrinos is given by,

$$\begin{aligned} P_{sol} &= 1 - 4U_{11}U_{11}U_{12}U_{12}\sin^2\left(\frac{\Delta m^2 L}{4E}\right) \\ &\quad -4U_{11}U_{11}U_{13}U_{13}\sin^2\left(\frac{(\Delta m^2 + \Delta M^2)L}{4E}\right) \\ &\quad -4U_{12}U_{12}U_{13}U_{13}\sin^2\left(\frac{\Delta M^2 L}{4E}\right). \end{aligned} \tag{9}$$

The extremely large L/E factor, as well as the range of Δm^2 considered here, enables one to simplify the above expression by carrying out averaging, which in terms of the mixing angles becomes

$$P_{sol} = 1 - 2c^2_{13}(s^2_{13} + s^2_{12}c^2_{12}c^2_{13}). \tag{10}$$

In the case of atmospheric neutrinos, the most important feature is the zenith angle dependence of the ratio of the observed neutrinos to the MC expectations. For example, the ratios in the case of electron neutrinos (R_e) and muon neutrinos (R_μ) are defined as follows,

$$R_e = P_{ee} + rP_{\mu e}, \tag{11}$$

$$R_\mu = P_{\mu\mu} + \frac{1}{r}P_{\mu e}, \tag{12}$$

where P_{ee}, $P_{\mu\mu}$ are the survival probabilities, $P_{\mu e}$ is the transition probability, and

$$r = \frac{N^o_\mu}{N^o_e} \tag{13}$$

is the expected ratio of fluxes without oscillations (where $N^o_{\mu(e)}$ is the initial flux of muon (electron) neutrinos). The ratio r varies from 1.6 (for sub GeV neutrinos) to 3.0 (for multi GeV neutrinos). Further, this ratio depends on the zenith angle also, varying from 1.6 for the horizontal neutrinos to 3.0 for the vertical ones. It is obvious that the uncertainties in r are bound to affect the calculation of the zenith angle dependence of R_e and R_μ. Since the purpose of the present communication is not to go into the extensive details of these aspects of data, therefore we would like to concentrate on that part of the data which is free from such uncertainties. Interestingly, it is to be noted that these uncertainties get highly suppressed for the case of high energy upward going neutrinos. It is also to be noted that the SK detector depicts better correlation for upgoing high energy neutrinos with the charged leptons produced in the detector. Further, the asymmetry A, defined as

$$A = \frac{U-D}{U+D}, \tag{14}$$

where U is the number of upward going events with zenith angles in the range $-1 < \cos\theta < -0.2$ and D is the number of downward going events with $0.2 < \cos\theta < 1$, is better defined for high energy neutrinos [22].

In view of the above mentioned reasons, it is desirable to consider the zenith angle dependence of R_e and R_μ for high energy upgoing neutrinos. The zenith angle dependence is generally expressed in terms of L/E. It is to be noted that for the vertically upgoing neutrinos, the length (L) traversed is $\sim$ 13000 km. For the SK geometry, the horizontal neutrinos correspond to those travelling $\sim$ 500 km, therefore, for the purpose of studying zenith angle dependence which does not involve uncertainty due to flux ratio r, it is desirable to avoid lengths $\simeq$ 1000 km.

Keeping in mind the mentioned constraints, the three flavor probability expressions P_{ee}, $P_{\mu\mu}$ and $P_{\mu e}$ required to evaluate R_e and R_μ can be expressed as follows. The electron neutrino survival probability is given by,

$$\begin{aligned} P_{ee} &= 1 - 4U_{11}U_{11}U_{12}U_{12}\sin^2\left(\frac{\Delta m^2 L}{4E}\right) \\ &\quad -4U_{11}U_{11}U_{13}U_{13}\sin^2\left(\frac{(\Delta m^2 + \Delta M^2)L}{4E}\right) \\ &\quad -4U_{12}U_{12}U_{13}U_{13}\sin^2\left(\frac{\Delta M^2 L}{4E}\right). \end{aligned} \tag{15}$$

Similarly the muon neutrino survival probability is given by,

$$
\begin{aligned}
P_{\mu\mu} &= 1 - 4U_{21}U_{21}U_{22}U_{22}\sin^2\left(\frac{\Delta m^2 L}{4E}\right) \\
&\quad -4U_{21}U_{21}U_{23}U_{23}\sin^2\left(\frac{(\Delta m^2 + \Delta M^2)L}{4E}\right) \\
&\quad -4U_{22}U_{22}U_{23}U_{23}\sin^2\left(\frac{\Delta M^2 L}{4E}\right). \qquad (16)
\end{aligned}
$$

Likewise, the transition probability is expressed as,

$$
\begin{aligned}
P_{\mu e} &= -4U_{21}U_{11}U_{22}U_{12}\sin^2\left(\frac{\Delta m^2 L}{4E}\right) \\
&\quad -4U_{21}U_{11}U_{23}U_{13}\sin^2\left(\frac{(\Delta m^2 + \Delta M^2)L}{4E}\right) \\
&\quad -4U_{22}U_{12}U_{23}U_{13}\sin^2\left(\frac{\Delta M^2 L}{4E}\right). \qquad (17)
\end{aligned}
$$

The factor L/E is quite large for the upgoing neutrinos considered here, therefore, for the purpose of calculations the above expressions get reduced to the following,

$$P_{ee} = 1 - 2U_{11}^2U_{12}^2 - 2U_{11}^2U_{13}^2 - 2U_{12}^2U_{13}^2, \qquad (18)$$

$$P_{\mu\mu} = 1 - 2U_{21}^2U_{22}^2 - 2U_{21}^2U_{23}^2 - 2U_{22}^2U_{23}^2, \qquad (19)$$

$$P_{\mu e} = 2U_{23}^2U_{13}^2 - 2U_{22}U_{12}U_{21}U_{11}. \qquad (20)$$

In Tables 1 and 2, we have presented the results of our calculations, for different mass hierarchies, incorporating the constraints due to the LSND results as well as due to the "key" features of SNP and ANP. To begin with, using the LSND probability given by the equation (6), we have calculated s_{13}. The values of ΔM^2 are constrained by the equation (8), the values of s_{23} by the equation (1) whereas the values of s_{12} used for calculations are fairly consistent with bimaximal mixing as well as with the analysis of Garcia *et al.* [26].

In Table 1, we have presented a representative sample of the calculations carried out in the case of "natural" mass hierarchy $\Delta m^2 \ll \Delta M^2$, with $\Delta m^2 = 10^{-4} eV^2$. The results corresponding to the hierarchy $\Delta m^2 < \Delta M^2$ are very much similar to those of $\Delta m^2 \ll \Delta M^2$, hence are not presented in the tables. It is interesting to mention that while carrying out the analysis of the range of s_{13}, using the LSND constraint, we find that s_{12} plays no role. However, because of the fact that P_{sol} depends on s_{12} values, it is not possible to consider values of $s_{12} < 0.6$. The fact that P_{LSND} is independent of s_{12} values can be easily

understood if one carefully examines the equation (6). Since $\Delta m^2 \ll \Delta M^2$, the first term of the equation (6) can be neglected and the contribution of Δm^2 from the second term can be ignored. The second and the third term can be combined, yielding in terms of mixing angles,

$$P_{LSND} = 4 s_{23}^2 c_{13}^2 s_{13}^2 \sin^2\left(\frac{\Delta M^2 L}{4E}\right) \; , \tag{21}$$

which has no s_{12} dependence. Therefore, while carrying out the calculations, we have considered the bimaximal mixing value of s_{12}. From Table 1, it is quite evident that when we scan the possible range of P_{LSND}, ΔM^2 and s_{23}, we get the range of s_{13} as $(0.07 - 0.29)$. From the table it should be noted that P_{sol} is very much in agreement with equation (3) for values of s_{13} only upto 0.18. Similarly, for the case of atmospheric neutrinos, R_e and R_μ also agree with the data [19] for the same upper limit of s_{13}. Therefore, when we consider the constraints imposed by SNP, ANP and the LSND experiment simultaneously, the range of s_{13} gets restricted to $(0.07 - 0.18)$. It needs to be added that for the region of L/E considered here, R_e and R_μ assume constant values because of averaging of various oscillatory terms in the probability expressions.

In Table 2, we have presented the results of the calculations corresponding to the "degenerate" case, *i.e.* where Δm^2 and ΔM^2 are of the same order of magnitude. As a first step, we have tried to find the largest possible value of Δm^2 which can fit the various constraints imposed by LSND, SNP and ANP. In this context, we find that a value of $\Delta m^2 > 0.085 \; eV^2$ is not able to fit P_{LSND}. It is also to be noted that the range of s_{23} is very much limited to values $\simeq 0.82$, which is almost near the largest value admitted by SK. Using these values of Δm^2 and s_{23}, we have calculated possible values of s_{13} as well as of s_{12} which fit the data. As is evident from the table, the values of s_{12} are also quite limited to the range $0.66 - 0.70$, whereas, s_{13} takes on much smaller values, $0.001 - 0.03$, as compared to the previous case where $\Delta m^2 \ll \Delta M^2$.

A closer scrutiny of our results reveals several points. It needs to be emphasized that the lower value of s_{13} found in the case $\Delta m^2 \ll \Delta M^2$, is in agreement with the value found recently by Garcia *et al.* [26], by carrying out a simultaneous analysis of solar, atmospheric and CHOOZ data. Interestingly, the range of s_{13} found by our calculations is in complete agreement with the range found by Akhmedov *et al.* [30], in the case of "LMA" solution of the SNP. Further, the upper range of s_{13}, surprisingly, is in agreement with that of CHOOZ constraint, however, the present upper limit, unlike the CHOOZ case, is scale independent. Similarly, in the second case where $\Delta m^2 \simeq \Delta M^2$, the calculated values of s_{13} correspond to the value of s_{13} found by Akhmedov *et al.* [30], in the case of "SMA" and "LOW" solutions. This agreement of s_{13} range found primarily from LSND data with other diverse analyses lends

strong support to the neutrino oscillations observed at LSND.

In the above mentioned tables, we have presented the results for two different mass hierarchies, e.g., $\Delta m^2 \ll \Delta M^2$ and $\Delta m^2 \simeq \Delta M^2$. In the latter case, we have already mentioned a specific value of Δm^2 which in fact is the highest value fitting the LSND data. In the first case, we have given a typical value, however our results are very much valid if Δm^2 corresponds to either the "LMA" or the "LOW" solutions to the SNP. The calculations also remain valid even if $\Delta m^2 \simeq 10^{-2}\ eV^2$.

In the tables, we have not included the results of the asymmetries calculated from equation (14). In case we impose the range of A within 1σ, e.g.,- 0.35 to - 0.25 [9], we find that the results corresponding to the degenerate case are within this range, however in the case $\Delta m^2 \ll \Delta M^2$ only values of $s_{13} \leq 0.1$ satisfy the above mentioned range of the asymmetry.

It is interesting to mention that the present calculations for both the mass hierarchies are fully compatible with the ν_μ disappearance observations at K2K. Although the K2K results show up only at 2σ level, still their compatibility with the present calculations supports the oscillation hypothesis.

In the present calculations we have not considered the downgoing atmospheric neutrinos, primarily because of the flux uncertainties associated with these neutrinos as well as because of the fact that high energy neutrinos do not oscillate for these lengths. However, it may be mentioned that for the low energy neutrinos $\Delta m^2 L/E$ can be ~ 1, in which case, the calculations become highly scale sensitive.

It is interesting to explore which additional features as well as mass hierarchies, apart from the "key" features of SNP and ANP, would be describable if one extends the present 3 active flavors to 3 active + 1 sterile scenario. The details of these will be published elsewhere.

After having found mixing angles which are compatible with the three neutrino experiments, it is desirable to examine the kind of texture specific mass matrices which would reproduce these mixing angles, as well as the possibility of CP violation in the leptonic sector. Regarding the mass matrices, some preliminary investigations have been carried out earlier [39], details in the present context would be published elsewhere. Regarding CP violation, assuming that δ is nonzero in the leptonic mixing matrix, one can estimate the order of CP violation, by calculating Jarlskog's rephasing invariant parameter J [40], through the relationship

$$|J| = 2 \times \text{ area of the unitarity triangle,} \tag{22}$$

for details we refer the reader to reference [41]. In analogy to the quark sector, J can be evaluated by using any of the six unitarity triangles in the leptonic sector, through equation (22). A nonzero area of any of the unitarity triangle

would imply CP violation, which we ensure by putting the constraint that the sum of the two sides of the triangle is always greater than the third side. Using this constraint J can be evaluated for any of the six unitarity triangles, however we will present the results corresponding to the unitarity triangle formed by considering the orthogonality of the I and II row of the mixing matrix. In figure 1, we have plotted the probability distribution of J by varying $s_{12},\ s_{23}$ and s_{13} within the ranges mentioned in the I row of the Table 3 and δ in the range 0^o to 180^o. Interestingly the histogram shows a sharp peak around $J = 2.126 \times 10^{-4}$, a corresponding guassian fit results in the range

$$J = (2.126 \pm 0.272) \times 10^{-4}. \tag{23}$$

The corresponding most probable range for δ is

$$\begin{aligned} \delta \ &= \ 1^o \text{ to } 10^o \qquad\qquad \text{(in I quadrant)}, \\ & \quad\ 170^o \text{ to } 180^o \qquad\quad \text{(in II quadrant)}. \end{aligned} \tag{24}$$

We have repeated the above procedure for the mixing angles listed in row II and III of Table 3. The corresponding J and δ are also listed in the table. In the case $s_{13} = 0.001 - 0.10$, we have mentioned for δ the value $90^o \pm 14^o$, however, δ can also exist with significant probability in the range 1^o to 5^o in the first quadrant and correspondingly 175^o to 180^o in the second quadrant.

It is interesting to consider the CP asymmetry in the case $\Delta M^2 \simeq \Delta m^2$. The CP asymmetry [21] is defined as follows

$$A_{CP} \ = \ \frac{P(\nu_\mu \rightarrow \nu_e) \ - \ P(\overline{\nu}_\mu \rightarrow \overline{\nu}_e)}{P(\nu_\mu \rightarrow \nu_e) \ + \ P(\overline{\nu}_\mu \rightarrow \overline{\nu}_e)} \simeq \ \frac{4\text{sin}^2\theta_{12}\text{sin}\delta}{\text{sin}\theta_{13}}\text{sin}\left(\frac{2\Delta m^2_{12}L}{4E}\right). \tag{25}$$

For the degenerate case, as well as using the LSND parameters L/E, we get a rough estimate of A_{CP} as follows,

$$A_{CP} \simeq \sin\,\delta. \tag{26}$$

As shown in the Table 3, for this case δ can take values 1^o to 10^o, therefore the asymmetry can be reasonable even for short baseline experiment like LSND, however in the case of LBL experiments the asymmetry can take large values. Similarly, in the case of natural mass hierarchy, the largest value of A_{CP} for LBL experiments, for example K2K experiments, can be given as

$$A_{CP} \simeq 0.4. \tag{27}$$

In conclusion, we would like to emphasize that we have carried out a simultaneous analysis, with three flavor oscillations, of the recently updated

LSND data along with the "key" features of solar neutrinos and atmospheric neutrinos. Interestingly, we find that there are three mass hierarchies which are able to fit the LSND data, the "averaged" solar neutrino probability and the zenith angle dependence for high energy neutrinos. The mass hierarchies found are $\Delta m^2 \ll \Delta M^2$, $\Delta m^2 < \Delta M^2$ and $\Delta m^2 \simeq \Delta M^2$, the latter being valid only for values of $s_{23} \simeq 0.82$. Out of these three mass hierarchies, the first and the second possibilities give similar results. In a particular case with $\Delta m^2 = 10^{-4} eV^2$, we find the range of s_{13} to be $0.07 - 0.18$. This range is in fair agreement with the "LMA" solution of Akhmedov *et al.* [30]. Further, the lower limit of this range is in agreement with the most recent and exhaustive analysis by Garcia *et al.* [26] based on a simultaneous analysis of solar neutrino data, atmospheric neutrino data and CHOOZ results. Furthermore, the range of s_{13}, for example $0.001 - 0.032$, found in the degenerate case corresponds to the values of s_{13} found by Akhmedov *et al.* [30] in the case of "SMA" and "LOW" solutions. These observations, in a way, show the compatibility of the oscillations observed at LSND with the solar and the atmospheric neutrino data. If in the case of atmospheric data, the up-down asymmetry is also included in our analysis, then the solution space gets restricted if the results of asymmetry are included upto 1σ level. However, if the results are included upto 2σ level, then most of the solutions remain valid. We have also made a rough estimate of CP violating asymmetries assuming the existence of CP violation in the leptonic sector. Interestingly, CP violation may be observable even at LSND.

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$P_{LSND}(\times 10^{-3})$	Δm^2	ΔM^2	s_{23}	s_{13}	P_{atm}	P_{sol}	R_e	R_μ
1.8	10^{-4}	0.40	0.82	0.07	0.33	0.50	1.00	0.56
2.0	10^{-4}	0.38	0.78	0.08	0.35	0.49	1.01	0.52
2.2	10^{-4}	0.35	0.74	0.10	0.36	0.49	1.01	0.51
2.4	10^{-4}	0.32	0.70	0.12	0.36	0.49	1.01	0.50
2.6	10^{-4}	0.29	0.66	0.15	0.35	0.48	1.01	0.52
2.8	10^{-4}	0.26	0.62	0.18	0.33	0.47	1.01	0.54
3.0	10^{-4}	0.23	0.58	0.23	0.30	0.45	1.00	0.58
3.2	10^{-4}	0.20	0.54	0.29	0.27	0.42	0.98	0.62

Table 1: Calculated values of $s_{13},\ P_{atm},\ P_{sol},\ R_e,\ R_\mu$ in terms of $s_{12},\ s_{23},\ \Delta m^2(eV^2),\ \Delta M^2(eV^2)$, with $s_{12} = 0.70$.

$P_{LSND}(\times 10^{-3})$	Δm^2	ΔM^2	s_{12}	s_{23}	s_{13}	P_{atm}	P_{sol}	R_e	R_μ
2.1	0.085	0.40	0.70	0.82	0.001	0.33	0.50	0.99	0.56
2.4	0.085	0.35	0.69	0.82	0.007	0.33	0.50	0.99	0.56
2.7	0.085	0.30	0.68	0.82	0.014	0.33	0.50	0.99	0.56
3.0	0.085	0.25	0.67	0.82	0.023	0.33	0.50	0.99	0.56
3.2	0.085	0.20	0.66	0.82	0.032	0.33	0.51	0.99	0.56

Table 2: Calculated values of $s_{13},\ P_{atm},\ P_{sol},\ R_e,\ R_\mu$ in terms of $s_{12},\ s_{23},\ \Delta m^2(eV^2),\ \Delta M^2(eV^2)$.

s_{12}	s_{23}	s_{13}	J	δ
0.5 - 0.7	0.54 - 0.82	0.001 - 0.05	$(2.104 \pm 0.262) \times 10^{-4}$	$\left(\begin{array}{c} 1^o \text{ to } 10^o, \\ 170^o \text{ to } 180^o \end{array} \right)$
0.5 - 0.7	0.54 - 0.82	0.001 - 0.10	$(2.382 \pm 0.079) \times 10^{-4}$	$90^o \pm 14^o$
0.5 - 0.7	0.54 - 0.82	0.05 - 0.15	$(1.179 \pm 0.650) \times 10^{-2}$	1^o to 180^o

Table 3: J and δ corresponding to different ranges of s_{13}.

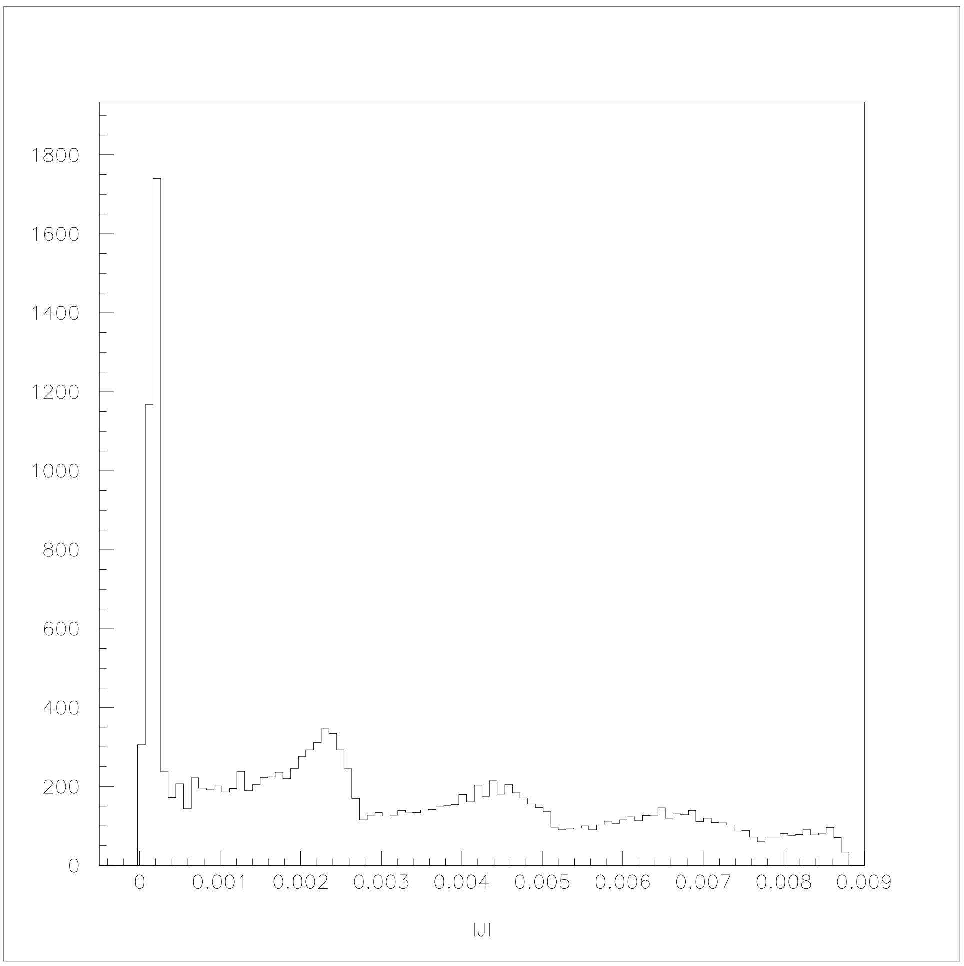

Figure 1: Histogram of $|J|$ plotted by considering the orthogonality relationship of I and II row of the mixing matrix.